Zadanie 2 seria 14:

$$\lim_{x \to \infty} \frac{1}{\sqrt{x}} \int_1^x \ln\left(1 + \frac{1}{\sqrt{t}}\right) dt = ?$$

Niech $f(x) = \int_1^x \ln\left(1 + \frac{1}{\sqrt{t}}\right) dt$

$g(x) = \sqrt{x}$.

Mamy $\lim_{x \to \infty} g(x) = +\infty$.

Zauważmy, że

$$f'(x) = \ln\left(1 + \frac{1}{\sqrt{x}}\right)$$

$$g'(x) = \frac{1}{2} x^{-1/2}$$

oraz

$$\lim_{x \to +\infty} \frac{f'(x)}{g'(x)} = \lim_{x \to \infty} \frac{\ln\left(1 + \frac{1}{\sqrt{x}}\right)}{\frac{1}{2} \frac{1}{\sqrt{x}}} = 2$$

Zatem z reguły de l'Hospitala

$$\lim_{x \to \infty} \frac{1}{\sqrt{x}} \int_1^x \ln\left(1 + \frac{1}{\sqrt{t}}\right) dt = 2$$